AQA

A-level
FURTHER MATHEMATICS
7367/3S

Paper 3 Statistics

Mark scheme

June 2024

Version: 1.0 Final

2 4 6 A 7 3 6 7 / 3 S / M S

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

No student should be disadvantaged on the basis of their gender identity and/or how they refer to the gender identity of others in their exam responses.

A consistent use of 'they/them' as a singular and pronouns beyond 'she/her' or 'he/him' will be credited in exam responses in line with existing mark scheme criteria.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M marks and is for accuracy
B	mark is independent of M marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
'their'	indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
sf	significant figure(s)
dp	decimal place(s)
ISW	Ignore Subsequent Workings

Examiners should consistently apply the following general marking principles:

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Q	Marking Instructions	AO	Marks	Typical Solution
1	Circles the 1st answer	1.1b	B1	4
	Question total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
2	Circles the 3rd answer	1.1b	B1	$1-e^{-0.7}$
	Question total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
3	Circles the 2nd answer	1.1b	B1	$\frac{10-3\sqrt{2}}{2}$
	Question total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
4	States both hypotheses using correct language using either $\lambda = 140$ or $\lambda = 20$ May use λ or μ	2.5	B1	$H_0 : \lambda = 140$ $H_1 : \lambda > 140$ $X \sim$ Po(140) P($X \geq 162$) = 0.037 $0.037 < 0.05$ Reject H_0 Sufficient evidence to suggest that the mean number of volcanic eruptions per week has increased.
	Uses Poisson model $X \sim$ Po(140) to calculate one of P($X \geq 162$) = **AWRT** 0.037 P($X \leq 161$) = **AWRT** 0.963 P($X > 162$) = **AWRT** 0.031 P($X \leq 162$) = **AWRT** 0.969 or P($X \geq 161$) = **AWRT** 0.044 **PI** by correct critical region	3.3	M1	
	Uses Poisson model to calculate P($X \geq 162$) = **AWRT** 0.037 or finds correct critical region $X \geq 161$	3.4	A1	
	Evaluates the Poisson model by correctly comparing their probability with 0.05 or 0.025 if a two-tailed test is performed or by correctly comparing 162 with their critical region	3.5a	M1	
	Infers H_0 rejected. **FT** comparison using their P($X \geq 162$) or P($X > 162$) with 0.05 or 0.025 if a two-tailed test is performed or their critical region which must correspond to an upper tail of the distribution. Condone Accept H_1	2.2b	A1F	
	Concludes, from a fully correct comparison, in context by referring to the mean number of volcanic eruptions per week. **OE** (Conclusion must not be definite, eg use of 'suggest', 'support')	3.2a	R1	
	Question total		**6**	

Q	Marking Instructions	AO	Marks	Typical Solution
5	Uses $\int \frac{x}{6}e^{\frac{1}{3}x}\,dx$ with any or no limits Condone missing dx	1.1a	M1	$\int_0^{\ln 27} \frac{x}{6}e^{\frac{1}{3}x}\,dx$ $=\left[\frac{x}{2}e^{\frac{1}{3}x}\right]_0^{\ln 27} - \int_0^{\ln 27} \frac{1}{2}e^{\frac{1}{3}x}\,dx$ $=\left[\frac{x}{2}e^{\frac{1}{3}x} - \frac{3}{2}e^{\frac{1}{3}x}\right]_0^{\ln 27}$ $=\frac{3}{2}\ln 27 - \left(\frac{9}{2} - \frac{3}{2}\right)$ $=\frac{3}{2}\ln 27 - 3$ $=\frac{3}{2}(\ln 27 - 2)$
	Uses integration by parts the correct way round to obtain $\left[kxe^{\frac{1}{3}x}\right] - \int ke^{\frac{1}{3}x}\,dx$ with any or no limits Condone missing dx **PI**	3.1a	M1	
	Obtains $\frac{x}{2}e^{\frac{1}{3}x} - \frac{3}{2}e^{\frac{1}{3}x}$ **OE** Components of the integrated function may be seen on different lines.	1.1b	A1	
	Substitutes limits of 0 and ln 27 and subtracts the correct way round in their integral of $\frac{x}{6}e^{\frac{1}{3}x}$ which must have at least two terms **PI** by sight of **AWRT** 1.94	1.1a	M1	
	Completes reasoned argument to obtain $\frac{3}{2}(\ln 27 - 2)$	2.1	R1	
	Question total		**5**	

Q	Marking Instructions	AO	Marks	Typical Solution
6(a)	States normally distributed **OE**	3.5b	B1	Assume that retirement ages are normally distributed.
	Subtotal		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
6(b)	States both hypotheses using correct language.	2.5	B1	H_0: $\mu = 29.5$ H_1: $\mu \neq 29.5$
	Obtains $\bar{x} = 30.42$ and $s^2 = 1.9525$ **OE** s^2 implied by $s = \frac{\sqrt{781}}{20}$ or **AWRT** 1.40	1.1b	B1	$\bar{x} = 30.42$ $s^2 = 1.9525$
	Attempts to calculate $t = \frac{\text{their } \bar{x} - 29.5}{\sqrt{\frac{\text{their } s^2}{5}}}$ Condone $z =$ or attempts to calculate $\text{their } \bar{x} - \text{their critical value} \times \sqrt{\frac{\text{their } s^2}{5}}$ or attempts to calculate $29.5 + \text{their critical value} \times \sqrt{\frac{\text{their } s^2}{5}}$	3.3	M1	$t = \frac{30.42 - 29.5}{\sqrt{\frac{1.9525}{5}}}$ $t = 1.47$ t_4 at 95% = 2.13 $1.47 < 2.13$ Do not reject H_0 Insufficient evidence to suggest that the mean retirement age is not 29.5
	Obtains t = **AWRT** 1.47 Condone $z =$ or obtains correct lower limit of confidence interval, **AWRT** 29.1 or obtains correct upper limit of critical region, **AWRT** 30.8	3.4	A1	
	Obtains correct critical value **AWRT** 2.13 or obtains p = **AWRT** 0.11	1.1b	B1	
	Evaluates t model by comparing their test statistic and their critical value or by comparing their p value with 0.05 or 0.1 if a one-tailed test is performed or by comparing 29.5 and their lower limit of their confidence interval or by comparing their $\bar{x}$ and their upper limit of their critical region	3.5a	M1	
	Infers H_0 not rejected. **FT** their comparison using the t model. Condone Accept H_0 or Reject H_1	2.2b	A1F	

	Concludes, from a fully correct comparison, in context by referring to mean retirement age. (Conclusion must not be definite, eg use of 'suggest', 'support')	3.2a	R1	
	Subtotal		**8**	

	Question total		**9**	

Q	Marking Instructions	AO	Marks	Typical Solution
7(a)	States $\frac{n^2-1}{12}$ **OE**	1.2	B1	$\text{Var}(X)=\frac{n^2-1}{12}$
	Subtotal		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
7(b)	States $Y=3X+10$ or Y replaced by $3X+10$	3.1a	M1	$Y=3X+10$ $\text{Var}(Y)=\text{Var}(3X+10)=3^2\text{Var}(X)$ $=3^2\times\frac{n^2-1}{12}$ $=\frac{3(n^2-1)}{4}$
	States or uses $\text{Var}(3X+k)=3^2\text{Var}(X)$ or $9\text{Var}(X)$ **PI** by $3^2\times$ their $\text{Var}(X)$	1.1a	M1	
	Completes reasoned argument to obtain $\frac{3(n^2-1)}{4}$ **OE** Must see Y replaced by $3X+10$ and $\text{Var}(3X+10)=3^2\text{Var}(X)$ or $9\text{Var}(X)$	2.1	R1	
	Subtotal		**3**	

	Question total		**4**	

Q	Marking Instructions	AO	Marks	Typical Solution
8	Uses correct formula for E(X)	1.1a	M1	E(X) = 7 × 0.4 + 13 × 0.35 + 17 × 0.1 + 21 × 0.15 E(X) = 12.2 P($X >$ E(X)) = P($X \geq 13$) = 0.35 + 0.1 + 0.15 = 0.6 P(E(X) < $X \leq 17$) = P($13 \leq X \leq 17$) = 0.35 + 0.1 = 0.45 P($X \leq 17 \mid X >$ E(X)) = $\frac{\text{P}(13 \leq X \leq 17)}{\text{P}(X \geq 13)} = \frac{0.45}{0.6} = 0.75$
	Obtains either P($X >$ E(X)) or P($X >$ E(X) ∩ $X \leq 17$) for their E(X)	1.1a	M1	
	Obtains P($X >$ E(X)) = 0.6 and P($X >$ E(X) ∩ $X \leq 17$) = 0.45 Condone incorrect E(X) that leads to these values	1.1b	A1	
	Uses conditional probability formula $\frac{\text{P}(X > \text{E}(X) \cap X \leq 17)}{\text{P}(X > \text{E}(X))}$	1.1a	M1	
	Completes reasoned argument to obtain P($X \leq 17 \mid X >$ E(X)) = 0.75 **CSO** Must see E(X) = 12.2 **OE** P($X >$ E(X)) = 0.6 **OE** and P($X >$ E(X) ∩ $X \leq 17$) = 0.45 **OE**	2.1	R1	
	Question total		**5**	

<table>
<tr><th>Q</th><th>Marking Instructions</th><th>AO</th><th>Marks</th><th>Typical Solution</th></tr>
<tr><td rowspan="3">9(a)</td><td>Calculates one correct row or column.</td><td>3.1b</td><td>M1</td><td rowspan="3"><table><tr><td></td><td>G</td><td>S</td><td>P</td></tr><tr><td>A</td><td>65</td><td>90</td><td>95</td></tr><tr><td>B</td><td>80</td><td>100</td><td>70</td></tr><tr><td>C</td><td>65</td><td>71</td><td>114</td></tr></table></td></tr>
<tr><td>Calculates two correct rows or two correct columns or one correct row and one correct column.</td><td>1.1b</td><td>A1</td></tr>
<tr><td>Obtains all the correct observed frequencies.</td><td>1.1b</td><td>A1</td></tr>
<tr><td></td><td>Subtotal</td><td></td><td>3</td><td></td></tr>
</table>

<table>
<tr><th>Q</th><th>Marking Instructions</th><th>AO</th><th>Marks</th><th>Typical Solution</th></tr>
<tr><td rowspan="8">9(b)</td><td>States both hypotheses using correct language. Variables need to be stated in at least the null hypothesis.</td><td>2.5</td><td>B1</td><td rowspan="8">H_0: There is no association between shop and rating
H_1: There is an association between shop and rating

<table><tr><td>E</td><td>G</td><td>S</td><td>P</td></tr><tr><td>A</td><td>70</td><td>87</td><td>93</td></tr><tr><td>B</td><td>70</td><td>87</td><td>93</td></tr><tr><td>C</td><td>70</td><td>87</td><td>93</td></tr></table>
$\sum\frac{(O-E)^2}{E}=$
$\frac{(65-70)^2+(80-70)^2+(65-70)^2}{70}+$
$\frac{(90-87)^2+(100-87)^2+(71-87)^2}{87}+$
$\frac{(95-93)^2+(70-93)^2+(114-93)^2}{93}$
$=17.60$

χ^2 cv for 4 df $=13.277$

$17.60>13.277$

Reject H_0

Sufficient evidence to suggest that there is an association between shop and rating.</td></tr>
<tr><td>Obtains correct expected contingency table for χ^2 model.
PI</td><td>3.3</td><td>B1</td></tr>
<tr><td>Attempts to calculate $\sum\frac{(O-E)^2}{E}$
Condone slips if intent clear
PI</td><td>3.4</td><td>M1</td></tr>
<tr><td>Obtains $\sum\frac{(O-E)^2}{E}$ = AWRT 17.6</td><td>1.1b</td><td>A1</td></tr>
<tr><td>Obtains correct critical value
AWRT 13.3
or obtains p = AWRT 0.001</td><td>1.1b</td><td>B1</td></tr>
<tr><td>Evaluates χ^2-test statistic by comparing their test statistic with their critical value or their p with 0.01</td><td>3.5a</td><td>M1</td></tr>
<tr><td>Infers H_0 rejected.
FT their comparison using the χ^2 model.
Condone Accept H_1</td><td>2.2b</td><td>A1F</td></tr>
<tr><td>Concludes, from a fully correct comparison, in context by referring to association between shop and rating. OE
(The conclusion must not be definite, eg use of ‘suggest’, ‘support’)</td><td>3.2a</td><td>R1</td></tr>
<tr><td></td><td>Subtotal</td><td></td><td>8</td><td></td></tr>
</table>

	Question total		**11**	

Q	Marking Instructions	AO	Marks	Typical Solution
10(a)	Obtains correct z or t value **AWRT** 1.70 **PI**	1.1b	B1	$z = 1.6954$ $10.2 \pm 1.6954 \times \frac{1.3}{\sqrt{250}}$ (10.06, 10.34)
	Uses formula for upper or lower limit of a confidence interval using their z or t value. **PI**	1.1a	M1	
	Obtains correct confidence interval **AWRT** 10.06 and 10.34	1.1b	A1	
	Subtotal		**3**	

Q	Marking Instructions	AO	Marks	Typical Solution
10(b) (i)	Explains that the null hypothesis is rejected because 10.36 is outside the confidence interval. Must see reference to 10.36 **FT** a correct conclusion for their confidence interval.	2.4	E1F	The null hypothesis is rejected as 10.36 is outside the confidence interval.
	Subtotal		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
10(b) (ii)	Uses formula for upper or lower critical value for the hypothesis test. **FT** their z value **PI**	1.1a	M1	$10.36 \pm 1.6954 \times \frac{1.3}{\sqrt{250}}$ (10.221, 10.499) $X \sim \text{N}\left(10.32, \frac{1.3^2}{250}\right)$ $\text{P}(X < 10.221) = 0.114$ $\text{P}(X > 10.499) = 0.015$ Power of test $= 0.114 + 0.015$ $= 0.13$
	Uses $X \sim \text{N}\left(10.32, \frac{1.3^2}{250}\right)$ model to find the probability of either their lower or upper tail or 1 – their P(Type II error). (must not use 10.2, 10.32 or 10.36) **PI**	1.1a	M1	
	Obtains correct power of the test **AWRT** 0.13	1.1b	A1	
	Subtotal		**3**	

	Question total		**7**	

	Paper total		**50**	